# meap

Michigan Educational Assessment Program

High School Test in Mathematics

Released Items

Spring 2002

2 Which type of graph would be **LEAST** effective for displaying population data over several decades?

 A line graph **B** circle graph

 C vertical bar graph **D** horizontal bar graph

4 What is the length of the side of the Stage 0 equilateral triangle if the perimeter of Stage 1 is 60 centimeters?
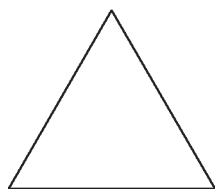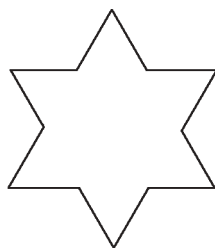

 Stage 0 Stage 1

 A 5 cm **B** 10 cm **C** 15 cm **D** 20 cm

7 If Paul, Quin, Ray, and Sam are seated at a circular table, what is the probability that Paul and Quin are seated **NEXT** to each other?

 A $\dfrac{1}{4}$ **B** $\dfrac{1}{2}$ **C** $\dfrac{2}{3}$ **D** $\dfrac{1}{1}$

12 What statistical measure would a store manager use to determine the best-selling item in the store?

 A mean **B** mode **C** range **D** median

14 The manager of Pete's high-rise apartment building assigns four parking spaces per floor. The parking spaces are numbered according to the following pattern:

1st floor	1100	1110	1120	1130
2nd floor	1200	1210	1220	1230
3rd floor	1300	1310	1320	1330
	• •			
	• •			
	• •			
15th floor	2500	2510	2520	2530

According to the pattern, what are the parking space numbers for the 11th floor?

A   1100   1110   1120   1130

B   11,000   11,100   11,200   11,300

C   21,000   21,100   21,200   21,300

D   2100   2110   2120   2130

15 Which of the following equations does this graph represent?
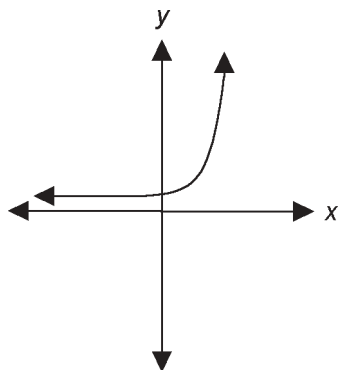
A $y = 3^x + 4$ **B** $y = 3x^3 + 4$

**C** $y = 3x + 4$ **D** $y = \dfrac{3}{x} + 4$

18 **(4 Points)**

The owner of an automobile dealership selected a sample from a population of 750 customers who received maintenance services last year. Due to time limitations, she sampled only 20 customers.

A The owner used the last 20 customers who received repair service as the sample. Explain why that sample would **NOT** be suitable.

B Describe a method of sampling 20 customers in a way that maximizes the probability the sample will fairly represent the population.

Explain your answers, including supporting calculations, tables, diagrams, charts, drawings/graphs in your answer booklet.

ANSWER THIS ITEM IN YOUR ANSWER BOOKLET.
NOTHING WRITTEN IN THIS TEST BOOKLET WILL BE SCORED.

MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM HST

SPRING 2002

SCORING GUIDE
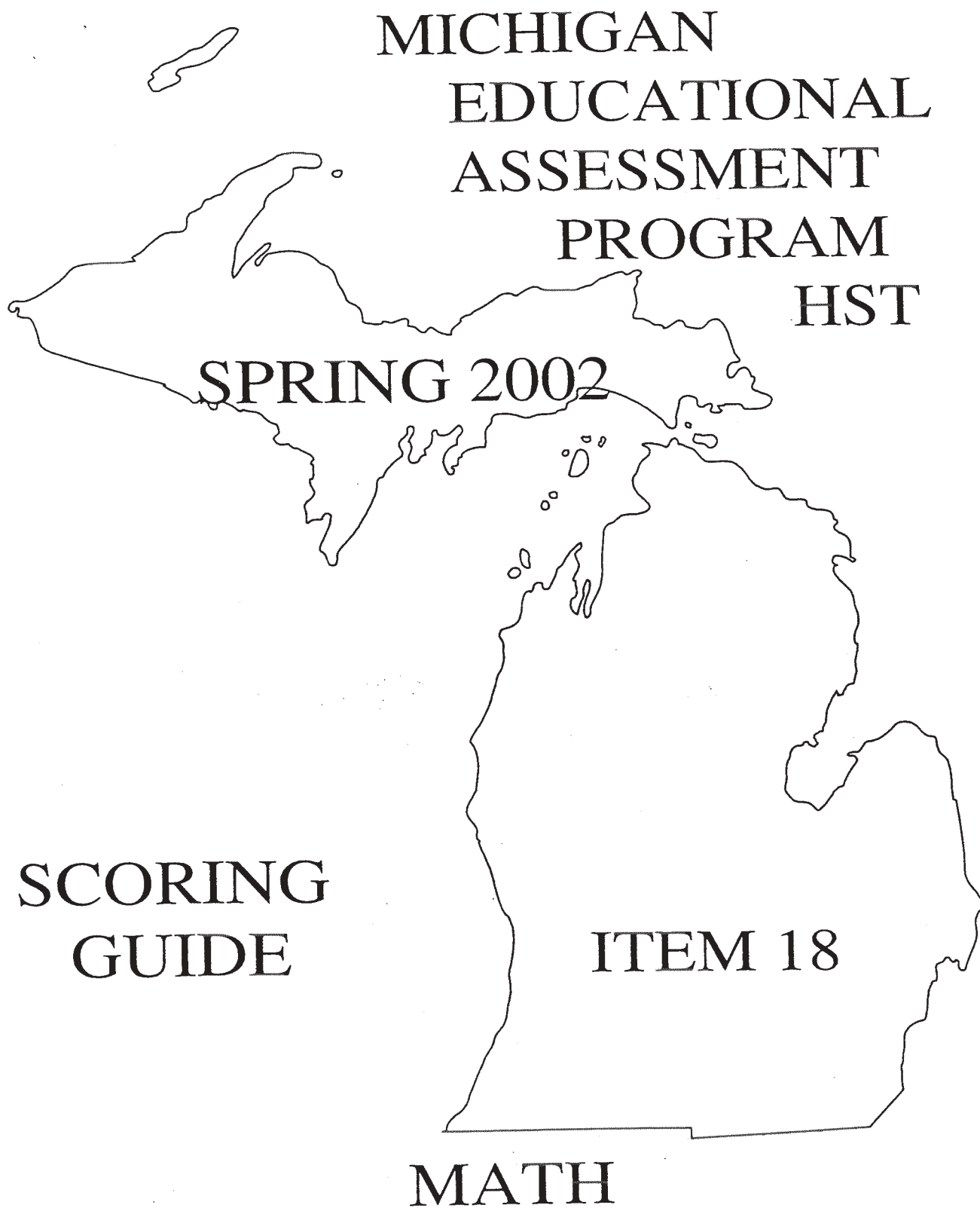
ITEM 18

MATH

MEAP HST 2002
Math
Item # 18
Scoring Rubric

A clear, correct answer for part A = 2 points
A clear, correct answer for part B = 2 points

A **4-point** response includes all of the following components:
- Demonstrates an understanding of sampling.
- **Part A:** Provides a clear, complete, valid explanation why the method is not suitable. The answer must include one of the following:
  a discussion of the *lack of randomness* in the sample with an explanation (e.g. It is not representative of the service over time because it will not reflect employees or service from months ago. If management changed a few weeks ago, the sample would only reflect service under the new management.).
  OR
  a discussion of the *sample size* being too small with an explanation (e.g. She only sampled 20 of the 750 customers. This is only 2.7% of the entire population of customers and therefore does not accurately represent the entire population.)
  OR
  an indication that there is a lack of randomness **and** sample size is too small.
- **Part B:** Provides a clear, complete description of a valid method for sampling 20 customers in a way that represents the population (e.g. Select every 37th customer; Sort the customers by phone number and select the first 20 customers whose phone numbers end in a 5; Put all the names in a hat and draw). The answer must be fully explained and demonstrate that the method will fairly represent the population.

A **3-point** response meets most of the criteria, but may do the following or similar:
- Demonstrates an understanding of sampling by clearly answering one part of the problem, but the answer for the other part is flawed or incomplete.

A **2-point** response meets some of the criteria, but may do <u>one</u> of the following or similar:
- Demonstrates some understanding of sampling by clearly answering one part of the problem, but the answer for the other part of the problem is incorrect, unclear or not attempted.
- Demonstrates some understanding of sampling, but the answers for both parts of the problem are flawed or incomplete.

A **1-point** response includes the following or similar:
- Demonstrates limited understanding of sampling by giving a flawed or incomplete answer for one part of the problem. The other part of the problem is either incorrect, unclear or not attempted.

A **0-point** response shows little or no understanding of the task.

Student Response 1

A This sample is not suitable because in a year's time, things change, like types of cars bought (newer models) A certain type or brand of car had a defect so most of the 20 people wanted it fixed, all of these people (if they serviced their car on time) would all have bought their car around the same time. Or she may have had a special going for her customers. Testing on 20 customers leaves the other 730 un accounted for.

B Starting with the first customer pick one every 37th person
So you would pick customers 1, 38, 75, 112, 149, 186, 223, ect.
This would ensure that you get people throughout the year with different cars and problems.

$$750 \div 20 = 37.5$$

* I rounded down to 37 because we are dealing with people.

$$1 + 37 = 38 + 37 = 75 + 37 = 112 + 37 \ldots$$

Scorepoint: 4

This response provides complete explanations of the lack of randomness (in a year's time things change, like types of cars bought...or...have had a special going) and sample size (testing on 20 customers leaves the other 730 unaccounted for) in Part A and a complete description of a method of random sampling (Starting with the first customers pick one every 37th person) in Part B.

Student Response 2

A

You can't sample only 20 customers out of 750. That's not a big enough sample. And she only sampled the last 20 instead of geting some costomers out of the begining and middle of the year.

B

The method she used isn't very affective because she only used 20 costumers. If she worted a good sample she should have used at Least a ¼ of the 750 customers. And she should have random picked them.

Scorepoint: 3

This response providews a partial explanation of sample size (not big enough sample) and lack of randomness (she only sampled the last 20 instead of getting customers out of the beginning and middle) in Part A and a partial description of a method of random sampling (used at ¼ of the 750... and ... random picked them) in Part B.

Student Response 3

A The sample would not be suitable because she did not have over half of the people she serviced. She picked her lst random 20 in which the probability of her finding better than average samples are poor.

B To maximize the probability you have to be mutually exclusive. That is through the year pick a numerous amount of samples and then break it down into 20 samples.

Scorepoint: 2

This response provides a partial explanation of sample size (she did not have over half the people she serviced) in Part A and a partial description of a method of random sampling (pick a numerous amount of samples and then break it down into 20 samples) in Part B.

Student Response 4

A The sample would not be suitable, because the population rate won't be as high. It is a big difference between 20 and 750, they would consider her project incomplete She would be 730 cars short.
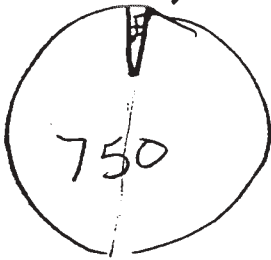
$$\frac{\begin{array}{r}750 \\ -\ 20\end{array}}{730 \text{ cars}} > \text{difference}$$

B

A method of sampling 20 customers in a way that maximizes the probability the sample will fairly represent the population, is by saying 20 customers came in 37 different Cars.

Scorepoint: 1

This response provides a partial explanation of sample size (big difference between 20 and 750) in Part A and no description of any method for sampling in Part B.

Student Response 5

A It wouldn't be suitable because there is 15000 customers but out of the customers only 20 will be sampled and its to many customers.

B 750 × 20 = 15000

Divide 15.000 from 750 leave you 20 customers that can be sampled

Scorepoint: 0

This response demonstrates no understanding in Part A and no description of any method for sampling in Part B.

19 **(4 Points)**

Toni's fully fueled race car completes one lap in 2 minutes 30 seconds but takes one second less for each successive lap as fuel is used. He makes a pit stop after every 10th lap to refuel. The pit stop lasts about 20 seconds. The race has 50 laps.

Assuming Toni doesn't make a pit stop on his final lap, how long will it take him to complete the race?

Explain your answer, including supporting calculations, tables, diagrams, charts, drawings/graphs in your answer booklet.

ANSWER THIS ITEM IN YOUR ANSWER BOOKLET.
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MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM HST

SPRING 2002

SCORING GUIDE
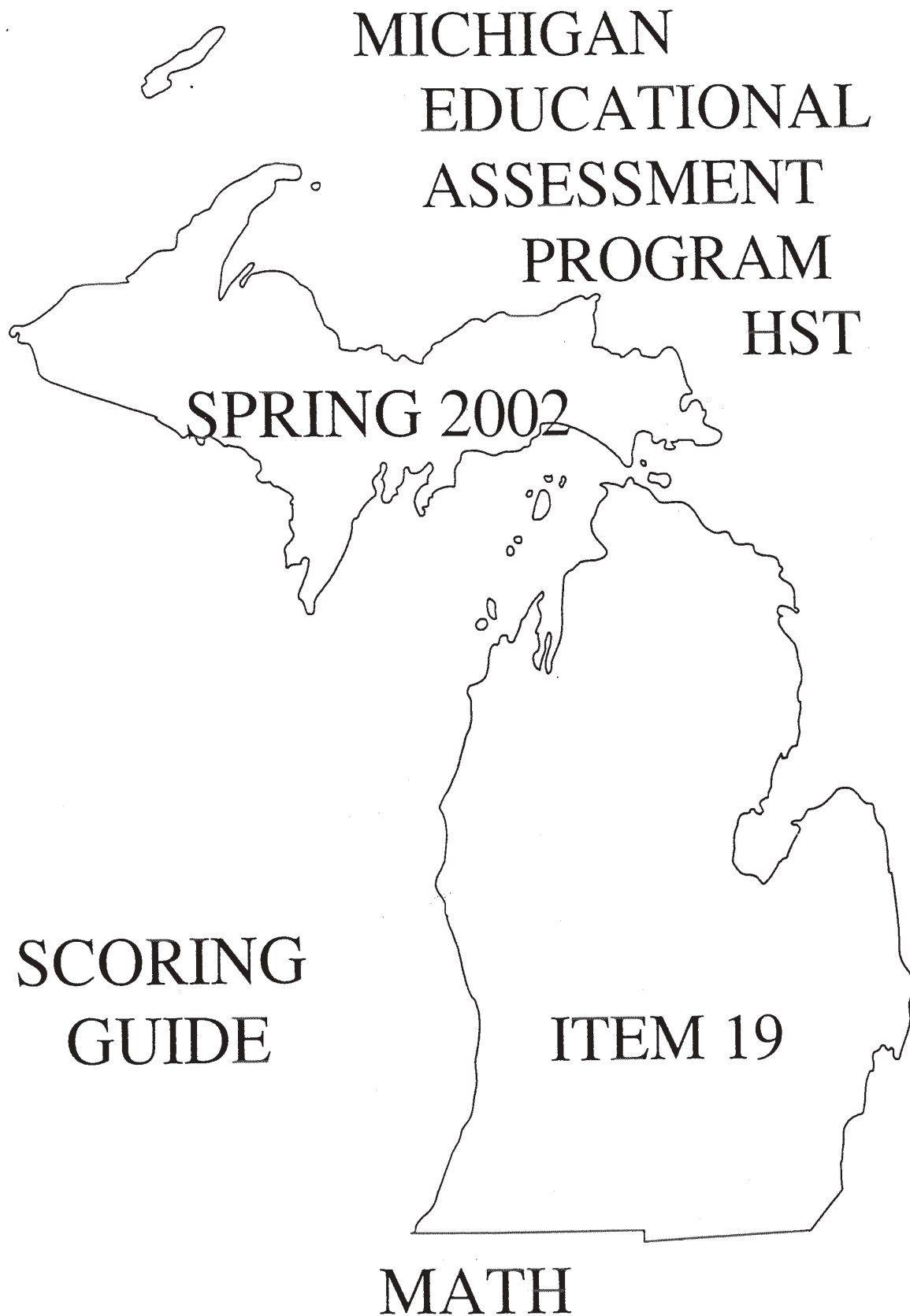
ITEM 19

MATH

MEAP HST 2002
Math
Item # 19
Scoring Rubric

A **4-point** response includes all of the following components:
- Demonstrates understanding of all four of the following components:
  1. There are five rounds of 10 laps (each of which begin again at 2:30).
  2. That 1 second is deducted from each lap.
  3. That there are a total of four pit stops (equaling 80 seconds to be added to the final time).
  4. That a conversion of time is involved.
- Correctly applies all four components to solve the problem, calculating a final answer of 2 hours 2 minutes 35 seconds. (OR: 122 minutes 35 seconds; 7355 seconds; 122.583 minutes; 2.04305556 hours; 2 hours 155 seconds; 120 minutes 155 seconds.)

A **3-point** response includes the following or similar:
- Applies a valid strategy to solve the problem, but makes 1 or 2 minor calculation errors (i.e. adds a 20 second pit-stop time after 50th lap) that result in an incorrect final answer.
  OR
  makes one significant error by neglecting one of the four components and this results in an incorrect final answer.

A **2-point** response includes the following or similar:
- Attempts to apply a valid strategy to solve the problem, but makes two significant errors or leaves parts of the problem incomplete.

A **1-point** response includes the following or similar:
- Attempts to apply a strategy to solve the problem, but the strategy is incomplete or unclear.   **OR**
- Attempts to apply a valid strategy but makes 3 significant errors.

Alternate 1: provides one of the correct answers with no work.

A **0-point** response shows little or no understanding of the task.

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Math
Item # 19
Scoring Rubric

- Clearly provides all supporting work and calculations, such as the following:

  2:30 + 2:29 + 2:28 + 2:27 + 2:26 + 2:25 + 2:24 + 2:23 + 2:22 + 2:21 + 0:20 = 24:35
  2:30 + 2:29 + 2:28 + 2:27 + 2:26 + 2:25 + 2:24 + 2:23 + 2:22 + 2:21 + 0:20 = 24:35
  2:30 + 2:29 + 2:28 + 2:27 + 2:26 + 2:25 + 2:24 + 2:23 + 2:22 + 2:21 + 0:20 = 24:35
  2:30 + 2:29 + 2:28 + 2:27 + 2:26 + 2:25 + 2:24 + 2:23 + 2:22 + 2:21 + 0:20 = 24:35
  2:30 + 2:29 + 2:28 + 2:27 + 2:26 + 2:25 + 2:24 + 2:23 + 2:22 + 2:21 = 24:15
  24:35 + 24:35 +24:35 +24:35 +24:15 = 2:02:35

Student Response 1

20 PIT

lap #	1	2	3	4	5	6	7	8	9	10	11	12	13
time(s)	150	149	148	147	146	145	144	143	142	141	150	149	148

20 PIT

lap #	14	15	16	17	18	19	20	21	22	23	24
time(s)	147	146	145	144	143	142	141	150	149	148	147

20 PIT

lap #	25	26	27	28	29	30	31	32	33	34	35	36	37	38
time(s)	146	145	144	143	142	141	150	149	148	147	146	145	144	143

lap #	39	40	41	42	43	44	45	46	47	48	49	50
time(s)	142	141	150	149	148	147	146	145	144	143	142	141

20 PIT

*The race will take him 122 min 35 sec to complete.

#1) add seconds together [7275 sec]
#2) add pit stop time [7355 sec]
#3) change to min [7355/60 = 122 min 35 sec]

Scorepoint: 4

This response provides the correct time of 122 min 35 sec and provides all four components.

Student Response 2

Laps	Time to complete lap (mins:secs)	(time × # of laps) total time to complete laps of this time
1, 11, 21, 31, 41	2:30	12:30
2, 12, 22, 32, 42	2:29	12:25
3, 13, 23, 33, 43	2:28	12:20
4, 14, 24, 34, 44	2:27	12:15
5, 15, 25, 35, 45	2:26	12:10
6, 16, 26, 36, 46	2:25	12:05
7, 17, 27, 37, 47	2:24	12:00
8, 18, 28, 38, 48	2:23	11:55
9, 19, 29, 39, 49	2:22	11:50
10, 20, 30, 40, 50	2:21	11:45

Race time = sum of total lap times

Race time = 117 mins + 4 mins 15 secs

= 121 mins 15 secs

Scorepoint: 3

This response provides an incorrect time due to one significant error (none of the four pit stops).

Student Response 3

TONI SHOULD COMPLETE THE RACE
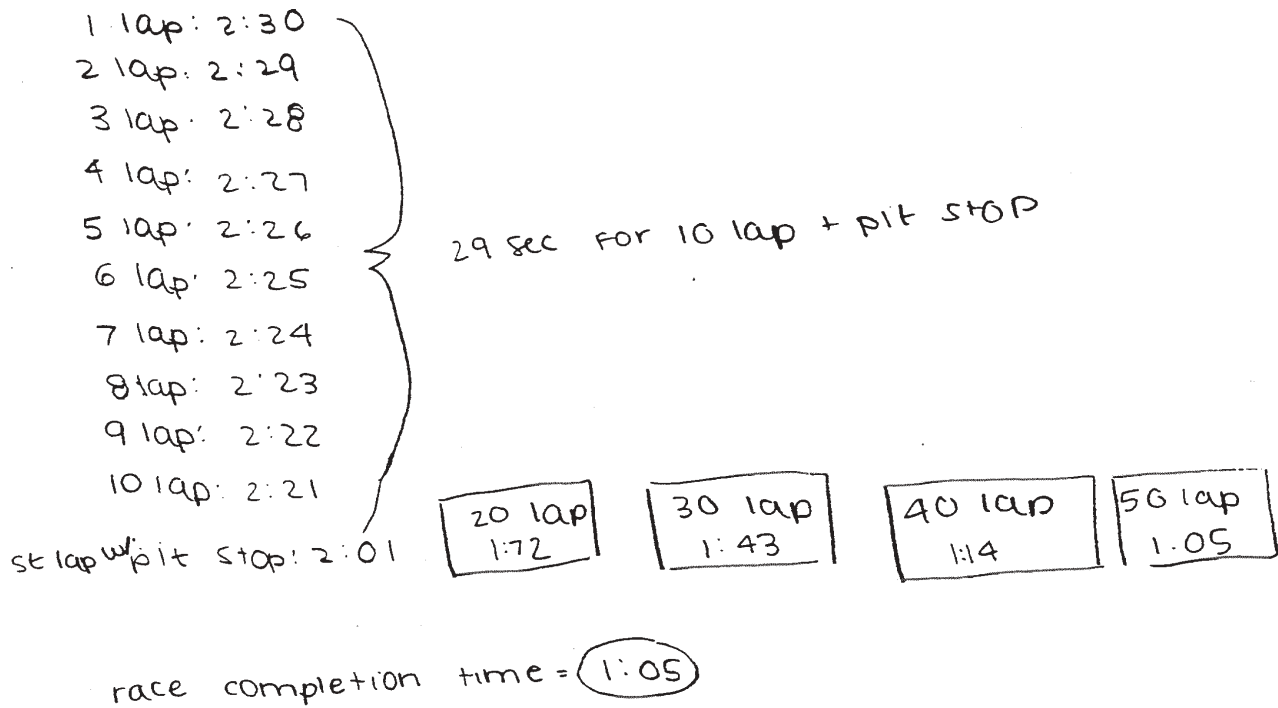Approximately 96 minutes 35 seconds

I drew out a table of each lap
time including the extra 20 sec.
for the pit stops on every
10th lap except the 50th where
TONI Doesn't make a pit stop.
I added all the times up to
get 96 minutes and 35 seconds
as a final time for Toni's
completion of the race

Lap	time
1	2:30
2	2:29
3	2:28
4	2:27
5	2:26
6	2:25
7	2:24
8	2:23
9	2:22
10	2:41
11	2:20
12	2:19
13	2:18
14	2:17
15	2:16
16	2:15
17	2:14
18	2:13
19	2:12
20	2:31
21	2:10
22	2:09
23	2:08
24	2:07
25	2:06
26	2:05
27	2:04
28	2:03
29	2:02
30	2:21
31	2:00
32	1:59
33	1:58
34	1:57
35	1:56

LAP	TIME
36	1:55
37	1:54
38	1:53
39	1:52
40	2:11
41	1:50
42	1:49
43	1:48
44	1:47
45	1:46
46	1:45
47	1:44
48	1:43
49	1:42
50	1:41

Scorepoint: 2

This response provides an incorrect time due to two significant errors (subtracts 1 second after each of the 50 laps and uses decimals to calculate time).

Student Response 4

1 lap: 2:30
2 lap: 2:29
3 lap: 2:28
4 lap: 2:27
5 lap: 2:26
6 lap: 2:25
7 lap: 2:24
8 lap: 2:23
9 lap: 2:22
10 lap: 2:21

29 sec for 10 lap + pit stop

5t lap w/ pit stop: 2:01

| 20 lap | 30 lap | 40 lap | 50 lap |
| 1:72 | 1:43 | 1:14 | 1.05 |

race completion time = (1:05)

Scorepoint: 1

This response provides an incorrect time due to three significant errors (does not start time over every 10 laps, subtracts and indicates 5 pit stops, and does not add all 10 laps together).

Student Response 5

2 mins 30 sec.

$$\begin{array}{r} 50 \\ \times 0.3 \\ \hline 115 \end{array}$$

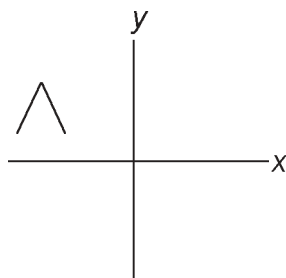$$\begin{array}{r} 115 \\ -60 \\ \hline 35 \end{array}$$

60 mins in an hr.
115 mins to do the race.

The race around the track 50 times with out stopping well take him about 2 hours. without any pit stops.

Scorepoint: 0

This response provides an incorrect time with no correct supporting work.

24 Which graph is a 180° rotation about the origin with respect to the given graph?

A

B

C

D
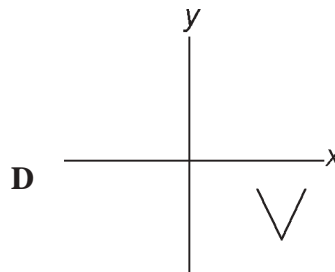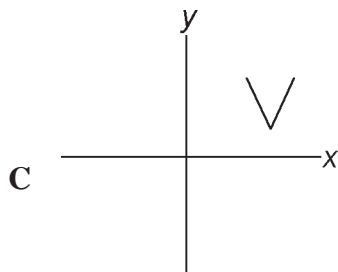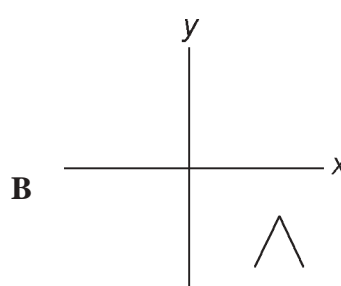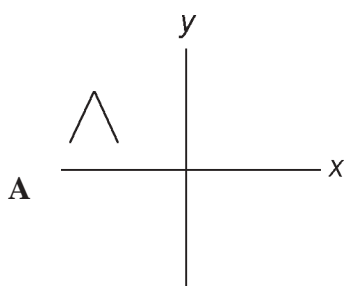
25 The area of a square made out of a length of yarn measures 225 square inches. How long is the yarn?

A 225.00 in   **B**   60.00 in   **C**   56.25 in   **D**   15.00 in

26 During a visit to his physician, John received a single dose of medication that decays at a rate of 20% per hour. The amounts of medication that remain in his body at the end of each hour for the first 3 hours are represented by the sequence 150, 120, 96 mg. Approximately how many milligrams of medication remained in his body after the first 6 hours?

A 25   **B** 30   **C** 50   **D** 90

31   John and Bill drove their truck to California. John drove for three straight hours averaging 55 mph. Bill then took over the driving for four straight hours averaging 65 mph. They continued this pattern until they covered 2125 miles. How long was their trip?

     A   14 hours   **B**   35 hours   **C**   44 hours   **D**   60 hours

32   Officials of the Park District plan to send a newsletter to the residents. Each newsletter weighs 1.8 oz. How many newsletters can they afford to send if they can spend up to $15,000?

Nonpresorted Bulk

First ounce or fraction of an ounce $0.296
Each additional ounce or fraction of an ounce $0.245

Weight Not Exceeding Ounces	Rate
1	$0.296
2	$0.541
3	$0.786

     A   27,726   **B**   28,153   **C**   30,487   **D**   50,675

34   Vinny divided his total paycheck as follows:

     $\frac{1}{2}$ into savings; $\frac{1}{10}$ for a payment; $48.00 for spending

What was the total amount of his paycheck?

   **A** $96   **B** $120   **C** $130   **D** $150

35   Heron's formula states that the area of any triangle with sides of lengths a, b, and c is:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where s is the semi-perimeter of the triangle, $s = \dfrac{a+b+c}{2}$

Use Heron's formula to find the area of a triangular plot of land with sides 9 meters, 11 meters, and 16 meters.

A $18\sqrt{7}$ m²   **B** $3\sqrt{14}$ m²   **C** $30\sqrt{15}$ m²   **D** $36\sqrt{22}$ m²

37   A can manufacturer plans to print the company name across the bottom of its cylindrical cans and also around the middle of the can. He discovered they could print the name, side by side, 7 times across the diameter of the bottom of each can. Approximately how many times could the name fit around the middle of the can?

A 14   **B** 22   **C** 35   **D** 49

38   A mountain bike costs $75 more than 3 times the amount a street bike costs. If the mountain bike sells for $1,500, which equation can be used to find the price of the street bike?

A $3x + 75 = 1500$ **B** $3x = 1500 + 75$

C $3(x + 75) = 1500$ **D** $3x - 75 = 1500$

39   In the parallelogram, if AB = BC = CD = DE, what is the ratio of the area of triangle *CDF* to the area of triangle *ABF*?
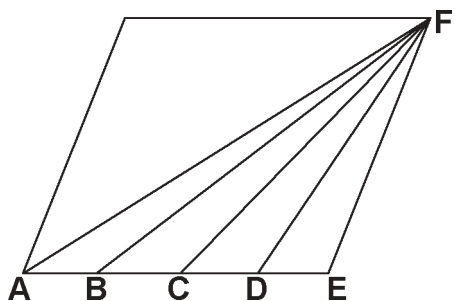
A $1:4$ **B** $1:2$ **C** $1:1$ **D** $4:1$

40 The first five terms of a sequence are 5, 11, 17, 23, 29,...
Identify the 20th term (a_{20}).

A 114   **B** 119   **C** 125   **D** 143

41 An economist is studying the supply and demand of a particular product. She determined that
demand for the product decreases as its price, P, increases according to the demand equation
$P = -\frac{2}{3}x + 30$. The supply of the product increases as its price increases according to the supply
equation $P = \frac{4}{3}x$. At what price, x, are supply and demand in equilibrium?

A 0   **B** 15   **C** 20   **D** 45

42 Which segment is the altitude of this pyramid?
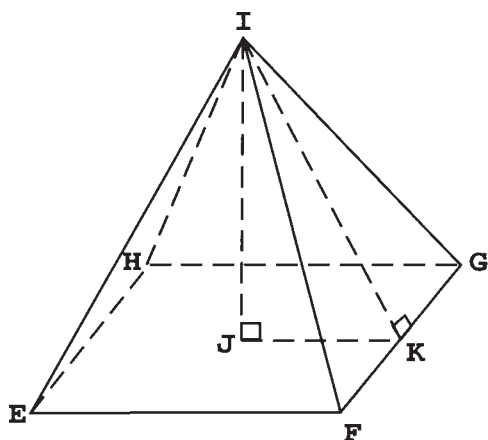
A $\overline{IJ}$ **B** $\overline{IG}$ **C** $\overline{IK}$ **D** $\overline{JK}$

43   (4 Points)

Solve the following problem:

Squares will be cut from the corners of a 10-inch by 6-inch rectangular piece of sheet metal in order to form an open-top rectangular box with a bottom area of 32 square inches.

A   Sketch a drawing of the rectangular sheet indicating where the cuts are to be made.

B   What size squares should be cut from the corners?

C   Give the computations used to find the size of the squares.

Explain your answers, including all supporting calculations, tables, diagrams, charts, drawings/ graphs in your answer booklet.

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SCORING
GUIDE
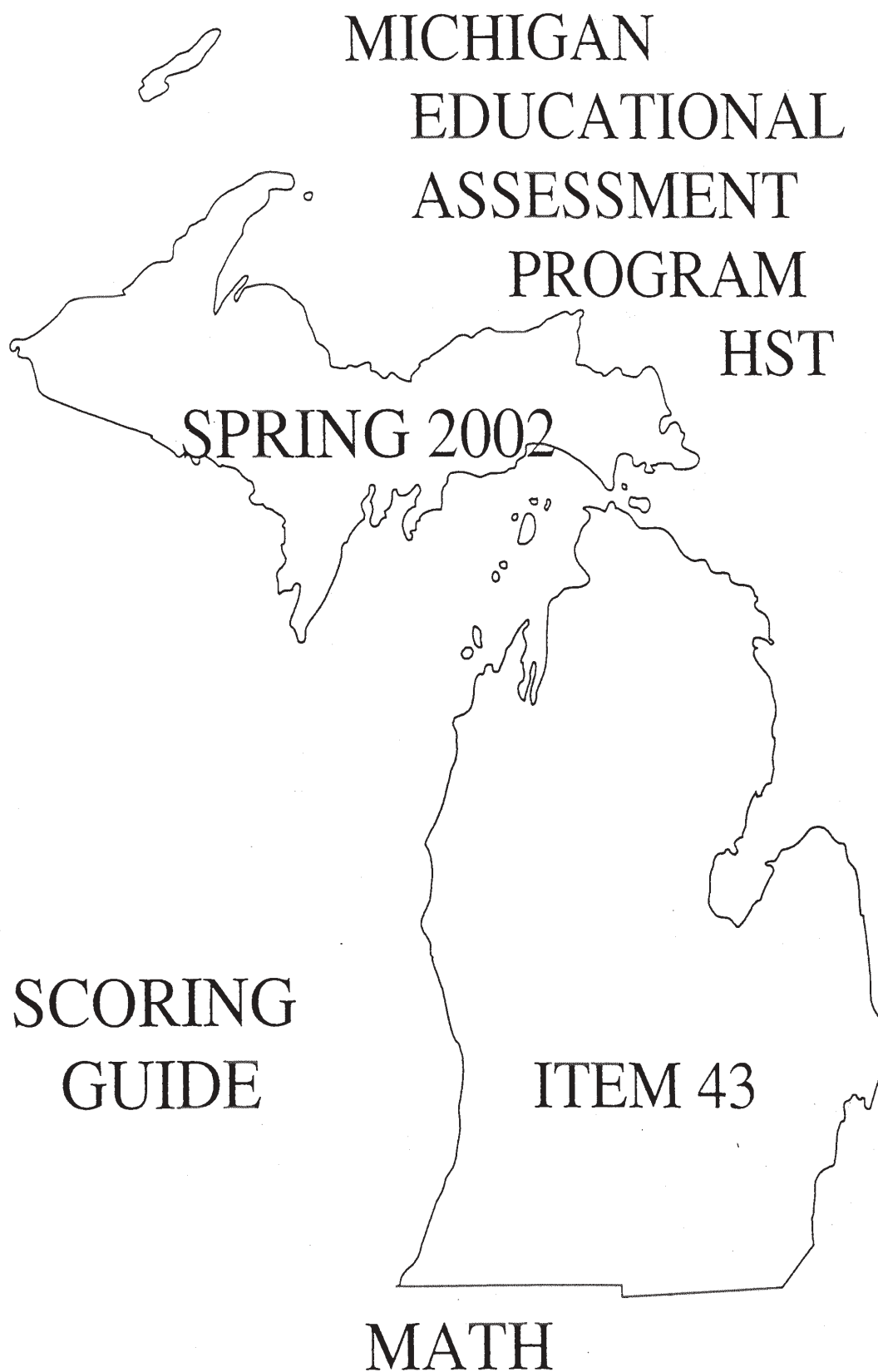
ITEM 43

MATH

MEAP HST 2002
Math
Item # 43
Scoring Rubric

Sample Response

A:

6 - 2s

s
s

← 10 - 2s →

Trial and error =
$$32 = 16 \times 2 \text{ (is not possible)}$$
$$= 8 \times 4 \text{ (works okay)}$$
short side cannot be 8, therefore

4

s
s

← 8 →
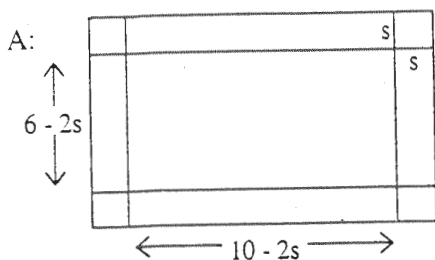
B: The squares are 1"x1"

C: $(6 - 2s)(10 - 2s) = 32$
    OR
Trial and error (see example on the right)
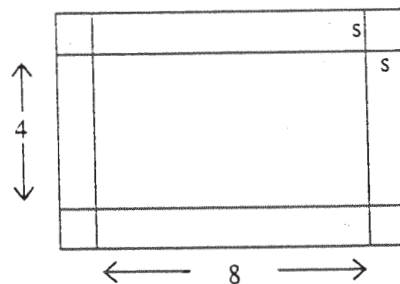
A **4-point** response includes all of the following components:
- A: Sketches a drawing of the box indicating where the cuts are to be made (**1pt**).
- B: Indicates that each square cut sh⸱ ⸱d be 1-inch by 1-inch (**1 pt**).
- C: Clearly provides all supporting work and explanations (e.g. $(6 - 2s)(10 - 2s) = 32$)
  (**2 pts**).

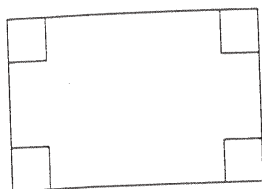A **3-point** response earns three of the four possible points.

A **2-point** response earns two of the four possible points.

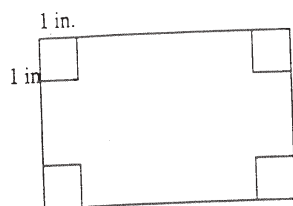A **1-point** response earns one of the four possible points.

A **0-point** response shows little or no understanding of the task.
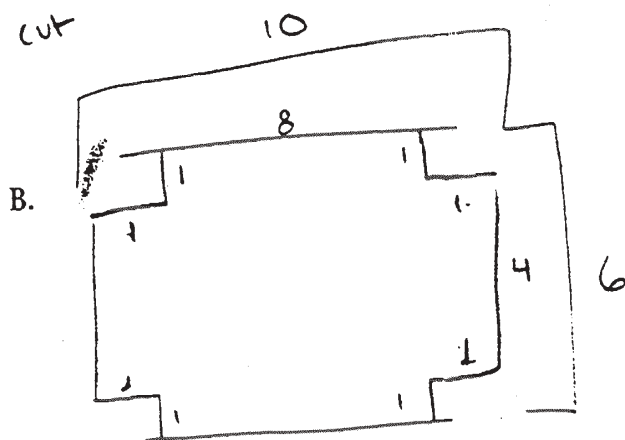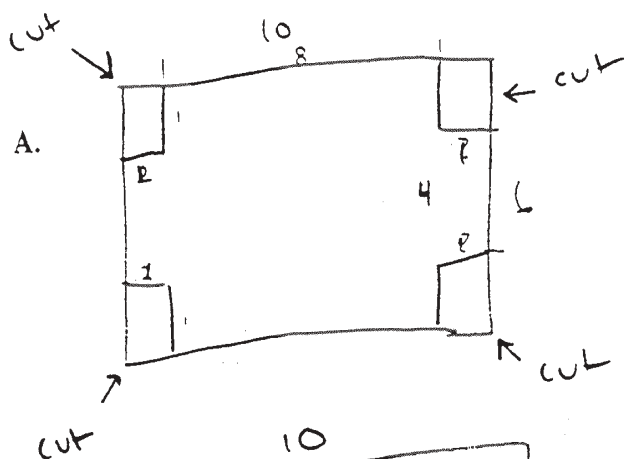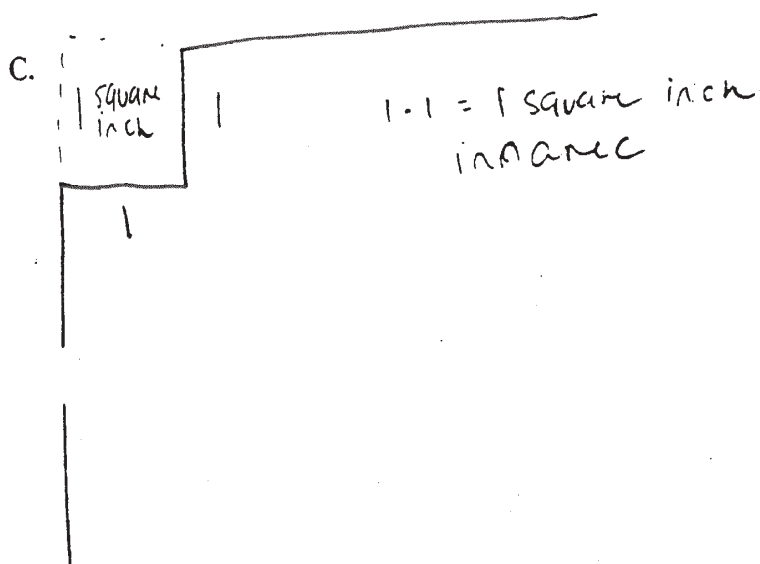
Additional Examples

1 point

2 points

1 in.

1 in.

Student Response 1

A.

B.

- The cuts should be 1 x 1 inch taken from each corner.

key

- - - - - = old lines before cut

C.

1 square inch
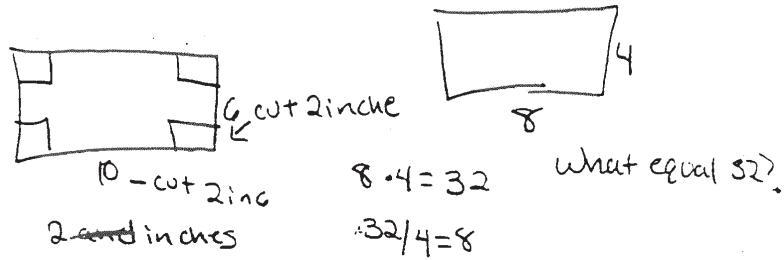
1 · 1 = 1 square inch in area

Scoreoint: 4

This response correctly sketches where the cuts are to be made, indicates the correct size squares and gives the computations used to find the correct size of the squares.

Student Response 2

A.

6 cut 2 inche

10 − cut 2 inc

2 end inches

$8 \cdot 4 = 32$

$32/4 = 8$

8

4

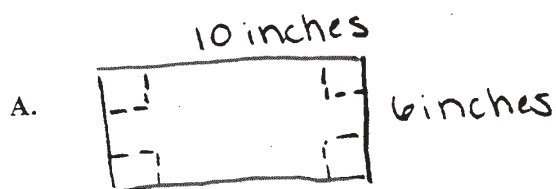what equal 32?

B. 2 inches off of each sid

C. What ='s 32

$8 \cdot 4 = 32$

Scorepoint: 3

This response correctly sketches where the cuts are to be made and provides the supporting work needed to find the correct size of the squares, but Part B indicates incorrect square sizes.

Student Response 3
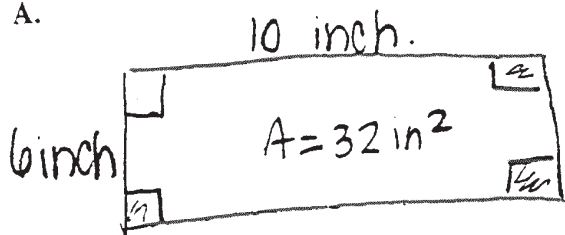
A.

10 inches

6 inches

B. 2 inches by 2 inches

c. Use 2 by 2 inch squares makes your square an 8 inches by 4 inches. This would give you an area of 32 inches.

Scorepoint: 2

This response correctly sketches where the cuts are to be made and provides partial supporting work (8 x 4 = 32). The size of the squares indicated in Part B is incorrect.

Student Response 4

A.

10 inch.

6inch   A=32 in²

B.

7inches

C. original area
A=LxW
= 6 x 10
= 60 in²

60in² - 32in² = 28 in²
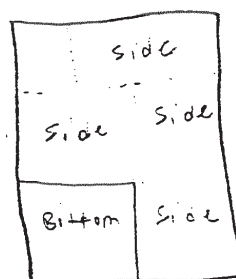
$\dfrac{28 in^2}{4} = 7 in$

Scorepoint: 1

This response correctly sketches where the cuts are to be made. An incorrect method is used in Part C and produces an incorrect size for the squares (7 inches) in Part B.

Student Response 5

A.

B.

C.

Scorepoint: 0

This response demonstrates no understanding of the item being tested.

Michigan Educational Assessment Program
Statewide Test Item Analysis
HST in Mathematics
All Students
Spring 2002

District: MICHIGAN DEPARTMENT OF TREASURY
Codes: District- 99999   School- 0000
Run Date: 08/06/2002

Multiple Choice

Percent Answering by Response

Item No.	Benchmark Code	A	B	C	D	Omit/Mult
Patterns, Relationships, and Functions						
04	2HS1	11	17	41*	30	0¤
14	1HS5	4	6	6	84*	0¤
26	1HS3	12	22	62*	4	1
31	2HS1	9	67*	15	8	1
40	1HS1	6	71*	15	6	1
Geometry and Measurement						
24	3HS3	7	13	26	53*	1
25	3HS2	22	27*	25	26	1
37	1HS7	42	31*	14	12	1
39	3HS2	16	26	47*	10	1
42	1HS2	70*	5	17	7	1
Data Analysis and Statistics						
02	1HS3	8	83*	4	5	0¤
12	2HS2	20	55*	16	9	0¤
32	2HS1	49*	14	23	13	1
Numerical and Algebraic Operations and Analytical Thinking						
15	2HS2	42*	15	33	10	1
34	1HS2	11	65*	13	9	1
35	1HS4	47*	15	21	17	1
38	1HS2	57*	12	20	10	1
41	2HS3	10	54*	24*	11	1
Probability and Discrete Mathematics						
07	1HS2	34	37	26*	3	0¤

Number of Students Included: 100526

Constructed Response

Item No.	Benchmark Code	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	A	B	C	D
Patterns, Relationships, and Functions														
19	#	17	8	14	11	10	10	12	1	9	1	0	0	7
Numerical and Algebraic Operations and Analytical Thinking														
43	#	30	4	22	4	6	3	2	2	16	1	0	0	10
Probability and Discrete Mathematics														
18	#	8	6	13	11	14	13	17	5	6	1	0	0	6

(Percent Receiving Number of Points / Percent Receiving Condition Codes)

Condition Codes for the Constructed-Response Items:

A   Off-topic
B   Illegible
C   Written in language other than English
D   Blank/refused to respond

The constructed-response items assess multiple benchmarks.

Using the Benchmark Codes

You can link the individual items to their corresponding benchmark in the *Michigan Curriculum Framework*, approved in 1996.

Each benchmark code contains four characters. The first character, an Arabic numeral, identifies the content standard under the specific strand. The next two characters represent the grade level column designation in the content standards documents (ES = Elementary School, MS = Middle School, and HS = High School). The number following these letters represents the specific benchmark in the column designated by the grade level.

EXAMPLE
An item with benchmark code 1MS2 under Geometry and Measurement is referring to content standard 1, Shape and Shape Relationships. Within that content standard, you need to look at middle school benchmark number 2, "generalize the characteristics of shapes and apply their generalizations to classes of shapes," to find the match.

CAUTION
Making inferences about students based on their answers to individual items is inadvisable due to the low reliability of single item measures. These data should only be used to make inferences about the performance of groups that are classroom size or larger.

Omit/Mult = Omits and Multiple Responses
¤ Number of students present rounds to zero